ARITHMETIC ⇒ ALGEBRA ⇒ CALCULUS.

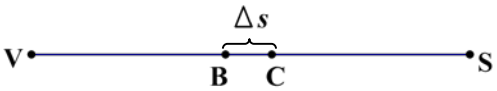
Understanding Instantaneous Rate of Change

You travelled a distance of 300 km and time taken is 3 hrs.

- At a point 'C' on the way you were stopped by a police officer for speeding.
- What speed were you traveling at the instant the police officer got you on his radar?

Consider the point C where the police officer stopped you. Take a point B close to C.

- Let the small distance BC = Δs
- Let the time taken to cover the distance Δs be Δt



- We get a more accurate measure of speed at C using the ratio $\frac{\Delta s}{\Delta t}$
- "*s*" stands for distance, "*t*" stands for time and "Δ" is used to indicate very small.

We can calculate the average rate of change; $\frac{\Delta s}{\Delta t}$, as '$\Delta$t' becomes AS SMALL AS POSSIBLE.

- We write 'Δt' tends to '0' using the notation: $\Delta t \to 0$.

Finding $\frac{\Delta s}{\Delta t}$ as $\Delta t \to 0$ is called finding a limit. This leads into Calculus.

We have a new operation to find $\frac{\Delta s}{\Delta t}$ as $\Delta t \to 0$

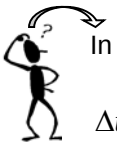
Find $\frac{\Delta s}{\Delta t}$ as $\Delta t \to 0$ is written as $\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$ - *Enjoy The Language of Mathematics - Calculus*

$\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$ is called the derivative or instantaneous change in '*s*' with respect to '*t*'

ARITHMETIC ⇒ ALGEBRA ⇒ CALCULUS.

CALCULUS

1. Elements are the Real Numbers & variables that stand for numbers
2. Operations combining real numbers & variables **…+, -, ×, ÷** , $\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$, $\frac{dy}{dx}$, $\int$ etc.
3. Relations**…< , =, >**



In Calculus, we have new operations,

$\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$: a limit

$\frac{dy}{dx}$: a derivative

$\int$ an integral.

$\frac{dy}{dx}$ is basically a division problem *with a twist*

$\int$ is an addition/ multiplication problem *with a twist*

The *twist* set in motion by $\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$

The honor for finding an efficient method to calculate the instantaneous rate of change in one variable '*y*' with respect to another variable '*x*' at a given point, goes to two famous mathematicians: Gottfried Leibniz & Isaac Newton

The language and methods developed by Gottfried Leibniz are used in the study of Calculus.

- Leibniz explicitly used the mathematical notion of a function, relating the change in one variable say '*y*' with respect to another variable say '*x*'.
- Calculus allows us to write real world problems in the mathematical language of equations. Algebra helps us to solve these equations.
- In order to study CALCULUS we begin with the study of RELATIONS AND FUNCTIONS.

Relation Mother → Child

Function: Child → Mother

Kamal's Potpourri

RELATIONS AND FUNCTIONS

A "relation" is just a connection between sets of information.

- Think of mothers and their children.
- In the pairing - we can call it an ordered pair (mother, child)

Let's suppose that there's a delivery guy bringing a prize for Caleb. Caleb has two brothers and all the delivery guy knows is that the prize is for the child. The mother takes the prize. She knows it is for a child. Who should she give it to? There are 3 different answers to the question “who is your child”. Confusing!

- Consider pairing with child the first variable, and mother second (child, mother).
- Caleb is told that his prize is with his mother. One answer to the question “who is your mother”. No confusion!

In the pairing (child, mother):

When the answer to a question is well defined, we have a relation which is function. Otherwise it is a relation but not a function.

- In mathematics we want one answer to one question

A RELATION WHICH IS A FUNCTION GIVES US
ONE ANSWER TO ONE QUESTION